647510 **MSC2MatC201x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
August/September -2020 [NEW COURSE]
Algebra – 2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

Q-1 Attempt any ***seven***: (14)

(a) Find [C : R].

(b) Define separable extension.

(c) Let F be a field with charF = p = 0. Does the polynomial $x^p - x$ in F[x], n > 1 has distinct roots ?

(d) State fundamental theorem of algebra.

(e) Define Aut(K), where K is a field.

(f) Express $x\backslash + x_2 + x^\wedge$ in the elementary symmetric functions in x_i, x_2, x_3.

(g) Define submodule.

(h) Define cyclic module.

(i) Let R be a ring with unity, and let M be an R-module generated by 0 = x € M. Then show that M is simple.

(j) Define completely reducible module

Q-2 Attempt any ***two***: (14)

(a) If L is a finite extension of K and if K is a finite extension of F then show that L is a finite extension of F.

(b) Show that K is a normal extension of F, if K is the splitting field of some polynomial over F.

(c) Find the degree of the splitting field of $x^4 + 4$ over Q.

Q-3 Answer the following. (14)

(a) State fundamental theorem of Galois theory.

(b) Find $G(K,$ Q) and $K_{G(K,Q)}$, where K = Q(^2).

OR

Q-3 Answer the following. (14)

(a) Let K be a normal extension of F, H be a subgroup of $G(K, F)$ and K_H be a fixed field of H. Then show that $[K : K_H] = o(H)$.

(b) Find the Galois group of $x^2 + 1$ over Q.

Q-4 Attempt any ***two***: (14)

(a) Let $(Nj)_{ieA}$ be a family of R-submodules of an R-module M. Then show that $P|_{ieA} N_i$ is an R-submodule.

(b) Let $A,$ $B,$ and C be R-submodules of an R-module M such that A D B. Show that A n (B + C) = B + (A n C).

(c) Let M be an additive abelian group. Show that there is only one way of making it a Z-module.

Q-5 Attempt any ***two:*** (14)

(a) Prove that the submodules of the quotient module, M/N are of the form U/N, where U is a submodule of M containing N.

(b) State and prove fundamental theorem of R-homomorphisms.

(c) Let M be a completely reducible module, and let K = M be a submodule of M. Show that M/K is completely reducible.

(d) Let M be an R-module. Then show that Hom_R(M, M) is a subring of Hom(M, M).

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